

Differential equation of S.H.M

① Differential equation of Simple Harmonic motion—
→ Consider a particle of mass m executing Simple harmonic motion. If y be the displacement of the particle from equilibrium position at any instant t , the restoring force F acting on the particle is given by

$$F \propto y \quad \text{or,} \quad F = -sy$$

Where s is the force constant of proportionality or, stiffness or, spring constant. The negative sign is used to indicate that the direction of the force is opposite to the direction of increasing displacement.

Force constant s is defined as the restoring force per unit displacement or,

$$s = \frac{F}{y} \quad \text{it's unit is Newton per meter.}$$

If $\frac{d^2y}{dt^2}$ is the acceleration of the particle at time t , then.

$$m \frac{d^2y}{dt^2} = -sy \quad \text{or,} \quad \frac{d^2y}{dt^2} + \frac{s}{m}y = 0$$

Substituting $\frac{s}{m} = \omega^2$, we get $\boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0}$ This is the general differential equation of \checkmark motion of a simple harmonic oscillator.